Quantum theory appendix 1: basic quantum theory

I would like to explain in more detail how this new approach to quantum theory works. To explain how this works I need to set out a framework of ideas which are sufficient to describe a physical system evolving in time, subject to probabilistic laws. This is not very complicated, and I will now explain the main ideas.

I will take an operational view of quantum physics, which is analogous to the operational view we took in earlier chapters with regard to space, time and motion. This means that we don't speak about how nature is, intrinsically,. We speak only about the interactions between a subsystem and its environment. This of course assumes we are dealing with a small subsystem of the universe, so this fits nicely into the paradigm of doing physis in a box.

From the operational point of view we describe experiments as ways of asking a question about a system. If the system is a particle, we might want to ask about its position or its velocity or its energy. Or the particle might be spinning, in which case we can ask about the direction of the axis around which it is spinning.

The system might be anything that can be described by asking questions. For example a fashion model who is wearing shoes, a dress and sunglasses. We then can ask questions about the colors of the different articles of clothes she is wearing.

Whether the system is a particle or a fashion model, we can describe any experiment as having three steps. First we prepare the system, which means we put it into a state where some of the questions have definite answers. Then we act on the system to transform it. Or we can just leave it alone, and see if it changes intrinsically in time-that is a kind of transformation too. The third step is to ask a question of it.

A fact about the world is that it may not be possible to ask all the questions of a system that are needed to fully describe it. For example you cannot find out simultaneously what is the position and the velocity of a quantum particle. Similarly in the world of quantum fashion models you may not be able to measure the color of someone's shoes and dress at the same time.

Position and velocity are thus called incompatible measurements for a quantum particle. Shoes color and dress color could be incompatible measurements for a quantum fashion model. When two measurements are incompatible you can choose which you ask about when making a measurement; you can choose either one, but only one.

The simplest systems allow only two possible answers to any question that might be asked. Suppose that the fashion palette this year has only two colors: black and red. You can choose to ask about the color of shoes, dresses or sunglasses, and in each case the answer is either black or red. Other systems allow more answers to questions, for example, there are years when there are four possible fashion colors: red, black, blue and green.

We want then to have a theory of fashion that will enable us to make predictions of the following kind: suppose that we prepare a friend with red shoes and then transform her by letting her spend an hour shopping at Prada. What is the probability that afterwards her shoes are still red rather than black? What is the probability that she emerges wearing a red dress? What is the probability that her sunglasses are black?

Quantum theory is a probabilistic theory so we will restrict ourselves to making probabilistic predictions, as quantum theory does. If we happen to be lucky and have a theory that makes definite predictions that will reveal itself because all the probabilities will be one or zero.

To summarize, we want a theory that can answer questions about the probabilities for answers to questions put to systems, after having been prepared and then transformed. There are a lot of theories that can be described in this general framework. What I want to do is describe physical principles which pick out quantum theory from all the other theories that can be described in this general way[1].

New systems can be prepared in several ways from old systems. We can put two systems together to make a new systems. We can assume that every question about system one is compatible with every question about system two so the questions we can ask involve a choice of one question about each system. We will assume also that the probabilities go get particular pairs of answers are just the product of the probabilities for the answers on each subsystem.

Or we can extract a subsystem from a larger system, in which case the questions that can be asked are a subset which concern only the system extracted.

So now suppose we have prepared and transformed a system and we want to be able to predict the probabilities to get answers for different questions that might be asked of it. A key question we want to ask is how much information we need to be able to make the best possible predictions of the probabilities for the answers to any question we might choose to make on the system. The maximal information we can have about a system, after preparation and transformation, but before we make a measurement of it, is captured by the notion of the state of the system. Given the state, we are able to infer probabilities for any question that may be posed of the system.

The amount of information needed to know the state is called the number of degrees of freedom. This can be taken literally, the bigger the number is, the more freedom the system has. For example, in quantum fashion, the degrees of freedom are the possible colors of the shoes, dress and sunglasses.

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[1] This kind of approach to quantum mechanics was pioneered by my colleague Lucien Hardy in 2001, for the technical details (not mentioned here) I am using a version due to another colleague Markus Mueller.

The state can be found by preparing many copies of the system, transforming them in the same way, and then making different measurements of the system, each many times, to build up statistics which give us the probabilities for different outcomes. So you prepare and transform our quantum fashion victim the same way many times. The first million times we measure the shoe color, which gives us probabilities for the two outcomes, red and black. The next million times we measure the probabilities for the dress color, the next million the color of the sunglass frames.

Once we have measured all the degrees of freedom, we have enough information to make predictions for any further question that might be asked, for example, heal hight or belt width.
So the more degrees of freedom the more measurements you have to make to know everything that can be known about the system.

You can increase the numbers of degrees of freedom several ways. Increasing the number of possible answers to each question-such as the number of possible fashion colors-increases the degrees of freedom. So does combing two systems, in which case the degrees of freedom of the two systems just multiply.

Let us consider two extreme cases to illustrate the concept of degrees of freedom. In classical physics there are no incompatible questions and every question can have a definite answer. In this case the number of degrees of freedom is just equal to the number of possible answers that any question can have, minus one. The simplest case is a bit of information: there are two possible states, YES and NO, and just one degree of freedom. The opposite case would be one in which there were an infinite number of questions that could be asked, and no correlations between them. No matter how many questions one knew the answer to, one could not predict the probabilities for the next question. In this case there are an infinite number of degrees of freedom.

This operational way of thinking about quantum physics was pioneered by Lucien Hardy in 2001 and developed by others. I find particularly useful a version developed by M and Mueller. They give a short list of assumptions about how the probabilities of which we have been speaking are affected when one combines two systems into one or prepares a system by projecting out a subsystem of a bigger system. I will not describe these here, but they are simple and natural. They then deduce quantum mechanics from these simple assumptions.

I noticed that a consequence of their work is that quantum physics can be understood to arise from a simple principle, which is that the number of degrees of freedom is as large as possible. More specifically, if we fix the number of possible answers and make some very reasonable assumptions, then what distinguishes quantum physics from other probabilistic theories is that the number of degrees of freedom is as large as possible.